Tuckswood Academy & Nursery

Calculations Guidance

September 2018

Contents

Rationale

'Mathematics is a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.' – National Curriculum

This policy has been written with core values of maths at its heart. Children need a conceptual understanding of maths, and this policy reflects that as it places emphasis on teaching conceptual methods, not procedural approaches. The policy is written to reflect the fundamental part that mathematics plays in the education of our children. The effective teaching of mental calculations has consistently been found to underpin success in written calculations and therefore fluency in mental calculation methods forms the central part of teaching.

In discussions, through research and ongoing CPD, we have found that there are not common approaches to teaching calculations. If we are consistent in the teaching of calculations then we should see a progression of the methods across the schools. This will benefit the children's learning when they make the transition between year groups and key stages, helping to overcome misconceptions.

The Norfolk calculations research, (Borthwick and Harcourt-Heath, 2006-2014) found that children who use traditional algorithms to calculate have far lower success than those who use strategies that are built on conceptual understanding. The research also found that children who used conceptually based methods outperformed those in Year 8 who use formal algorithms. Therefore to support mental and written calculations children are to be taught using a Concrete, Pictorial and Abstract (CPA) approach using representations in all year groups. This will develop their relational understanding of new concepts. As the children's understanding of mental and informal methods is defined so too are their abilities to solve problems, because the whole point of learning mathematics is to be able to solve problems.

Leaning rules and facts is important, but they are the tools with which children learn to do maths fluently. They are not maths itself. - (McClure 2013)

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects. The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

In addition all children will:

- develop a positive and enthusiastic attitude towards mathematics that will promote confidence and carry them throughout their lives.
- have a conceptual understanding of methods rather than a set of memorised procedures.
- use mathematical vocabulary correctly to communicate ideas.
- develop their relational understanding of new concepts, making connections through a CPA approach.
- demonstrate fluency in mental and written calculations.
- from EYFS to KS3 be secure in concepts and not moved into the next stage of learning before they are ready.
- be given opportunities to use and apply calculations in cross curricular mathematics.

Structure of the Policy

This calculation policy is structured in levels of progression from EYFS through to at least Key Stage 3. Each child should follow these stages of calculation for each of the four operations building on experiences and understanding. Children should progress at their own natural pace and should only move on once they show a secure readiness do so. The new National curriculum references 'Formal Methods of Calculation' – Children should only move onto formal methods of calculation once they reach full conceptual understanding of each operation through a CPA approach. This will vary from child to child.

Procedural V Conceptual

By definition, procedural methods relate 'to an official way of doing something' whereas conceptual methods build on a concept, a deep understanding, of the chosen task or calculation being explored. For children to succeed in maths, they need to be explicitly taught, shown, modelled and explore conceptual methods, gaining a full understanding before moving onto procedures.

> *'teaching formal written methods is inappropriate until children have made connections and developed their understanding of the number system'*
>
> *'…there is no case for introducing children to vertical layouts before they have developed a basic understanding of place value.'*
>
> *'Early introduction of vertical layout can be positively harmful.'*
>
> Derek Haylock and Anne Cockburn (1989), Understanding Early Years Mathematics,

Taking the rationale of the main policy and this theory into consideration then the teaching of formal (procedural) methods should only be taught when:

- The child has shown a sound understanding of place value and the number system.
- The child has shown a secure and conceptual understanding of informal methods of calculation
- The child has a secure understanding of their times tables up to 10x10.
- The methods are taught as a progression from informal methods, not as the default method.

If a child demonstrates misconceptions in the use of formal algorithms, then revert back to the progression of informal methods within the calculations policy to ensure that they are confident in calculation.

Concrete, Pictorial and Abstract

Concrete	Pictorial	Abstract
A new skill or idea is first introduced with the help of real objects. This manipulation of objects, perhaps combining two sets of objects for adding, is what underpins understanding.	During this phase the children's experience of objects means they can draw a representation of them rather than hold them. For example crossing out items to take away.	With solid and consistent understanding children can rely on mathematical notation to express their work. For example: 26-12=14

Language

Children who learn to explain why something makes sense and reason through their mathematical explanations will develop a deeper knowledge that will result in long term understanding. We want our children to be able to explain their methods confidently using the correct vocabulary. It is essential that all children are exposed to and supported in developing quality and varied mathematical vocabulary. This understanding will consequently support children in their ability to access problems, present mathematical conjecture and justification through reasoning.

Terminology	**Language/vocabulary that children need to be confident in using**
addition	add addition more plus increase and make sum total altogether score double half how many more is… than…? how much more is…?

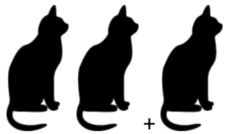
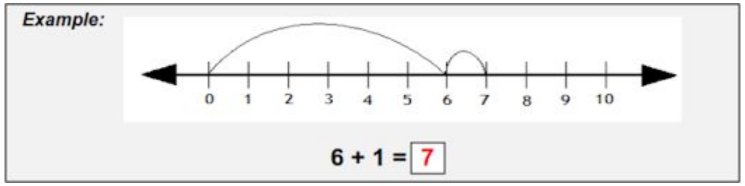
Terminology	**Language/vocabulary that children need to be confident in using**
subtraction	subtract take away minus decrease leave left over how many have gone one less, two less how many fewer difference between leave is the same as inverse

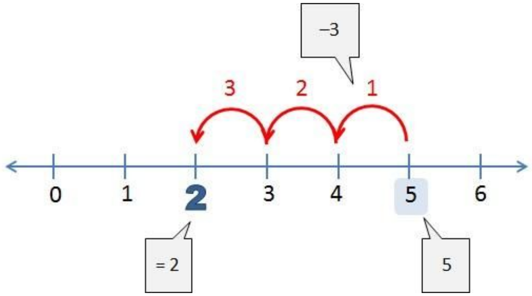
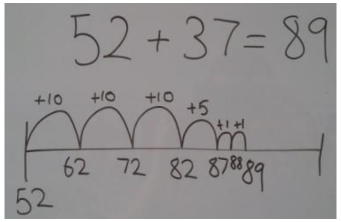
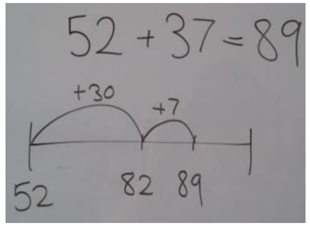
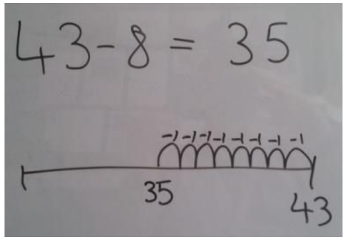
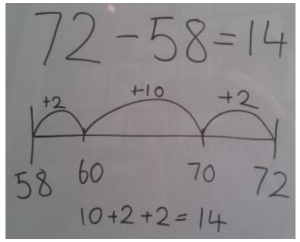
Terminology	**Language/vocabulary that children need to be confident in using**
multiplication	lots of groups of times multiplication multiply multiplied by multiple of product once twice times as (big, long, wide, and so on) repeated addition array row

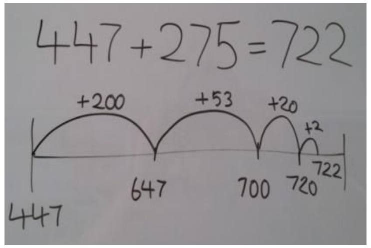
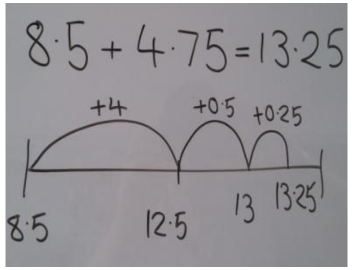
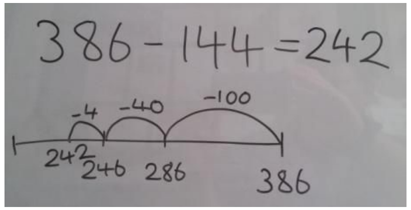
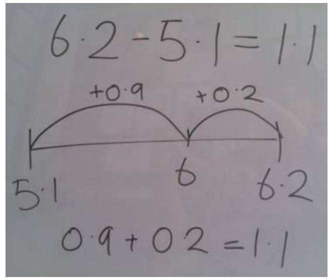
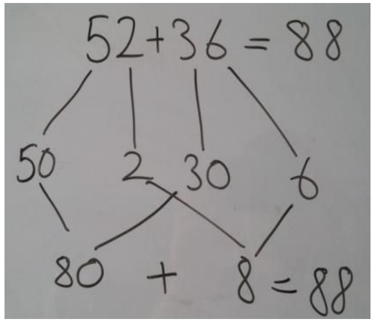
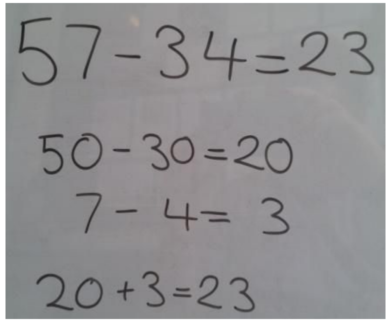
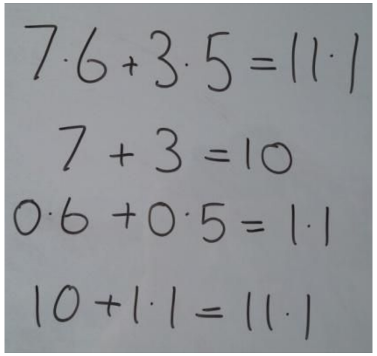
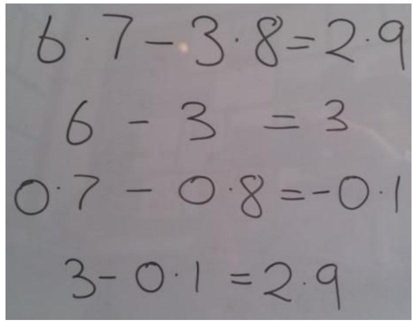
Terminology	**Language/vocabulary that children need to be confident in using**
division	halve share, share equally one each, two each, three each etc… group in pairs, divided by divided into divisible by remainder left over factor quotient inverse

Addition and Subtraction

- Addition and subtraction should be taught alongside each other, not as separate areas of maths.
- Green stages are conceptual stages, essential to a full, embedded understanding. Blue stages indicate procedures, only to be taught once full conceptual understanding is achieved.
- The new Key Stage 2 SATs require children that demonstrate the highest attainment and progress to show the ability to understand and use the formal algorithms.
- For some children year 5 will be the right time to introduce procedures, many others will leave year 6 without needing to 'move on' to these methods. It is for teachers to determine the right time with reference to this policy.

Stage	**Skill, concept or approach**	**Examples**
Development of Early Skills	Using real life examples and a variety of practical resources children will develop a sense of addition and subtraction. Counting activities will form the backbone of this and children will be encouraged to use correct mathematical language and record their results in a variety of ways. These may include informal marks and diagrams as well as digits.	There are two cats in the garden and one more comes along. How many now?  + =3 Three cats in the tree and one runs off. How many are left?  3-1=2
Number Lines	Start adding and subtracting using a number line. With subtraction you can count back to take away or count up to find the difference.	

		
Empty Number Lines **2 digit numbers**	Later switch to an empty number line to increase flexibility, speed and to allow demonstration of understanding. Reduce the number of jumps by increasing their size. Again there are two ways to approach a subtraction.	   

Empty Number Lines **3 digit numbers and decimals**	Empty number lines are suited to problems involving three digit numbers and decimals.	
Partitioning	Partition numbers into tens and ones. This is perhaps a faster strategy but is no more or less effective than a number line.	
Partitioning with large numbers	Partitioning is suitable for larger numbers too. With subtraction you will sometimes have negative numbers to contend with in the final reckoning.	

<table>
<tr><th colspan="3">The most important foundation to a sound understanding of a formal algorithm in addition is centred around place value. If a child does not have a full understanding of place value then they are not ready for this stage.</th></tr>
<tr>
<td>Column method with partitioning</td>
<td>This represents a step on from partitioning method of addition as it is the first time that children will see a vertical layout.</td>
<td>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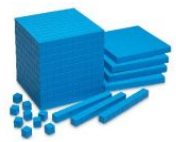
This stage should not be rushed as children need to comprehend that…

372 + 247 = 300 + 200 + 70 + 40 + 2 + 7 = 500 + 110 + 9 = 619

At this stage, the children will be inclined to add the hundreds first, then the tens before adding units. As this will have been the order of calculation when they have added by partitioning. When it is appropriate, the teacher should move them towards adding the units first, so that they are prepared for stages 2 and 3.



For calculating 372 + 247 =

<u>Stage 1:</u>

300 + 70 + 2

200 + 40 + 7

500 + 110 + 9 = 619

<u>Stage 2</u>:

  3 7 2

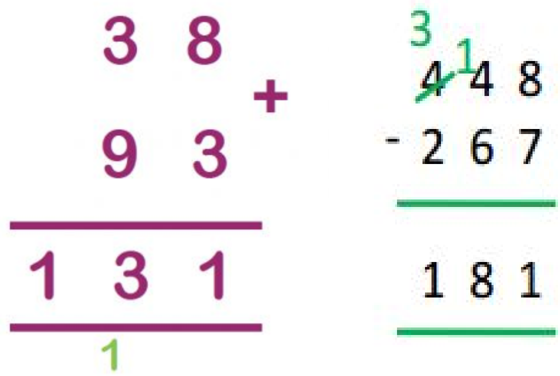
+ 2 4 7

     9 (2+7)

  1 1 0 (70 + 40)

  5 0 0 (300 + 200)

  6 1 9
</td>
</tr>
</table>

		452 – 361 = 400+50+2 300+60+1 100–10+1 = 91 1452 – 507 = 1000+400+50+2 0 +500+ 0+7 1000-100+50-5 = 945
Formal column method	This procedure represents the last step for addition and subtraction. Here, children are taught to borrow and to carry.	

Multiplication and division

- Multiplication and division should be taught alongside each other, not as separate areas of maths.
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- The new Key Stage 2 SATs require children that demonstrate the highest attainment and progress to show the ability to understand and use the formal algorithms.
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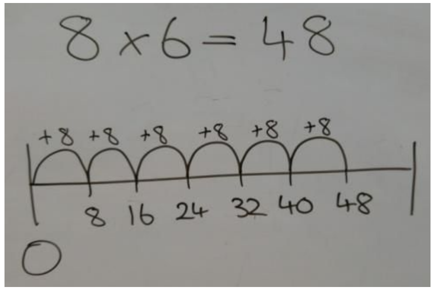
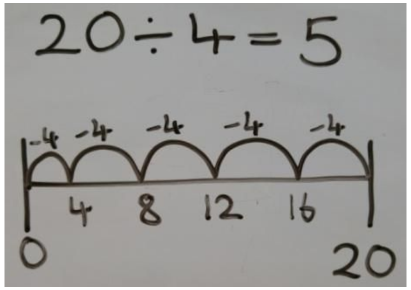
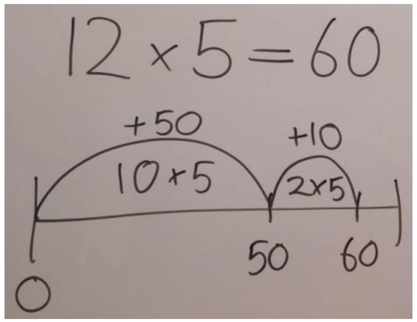
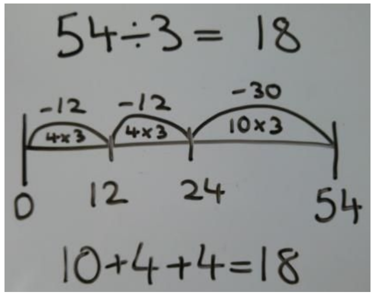
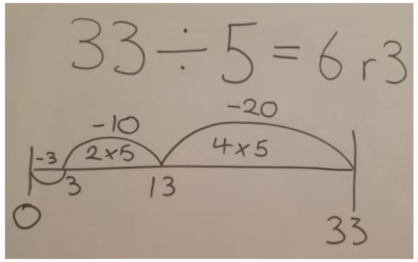
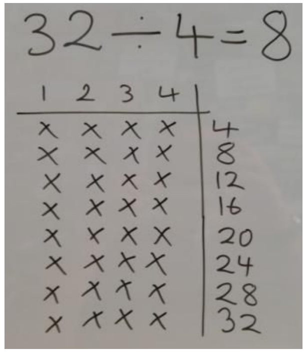
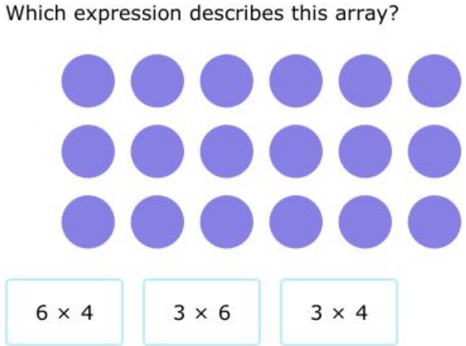
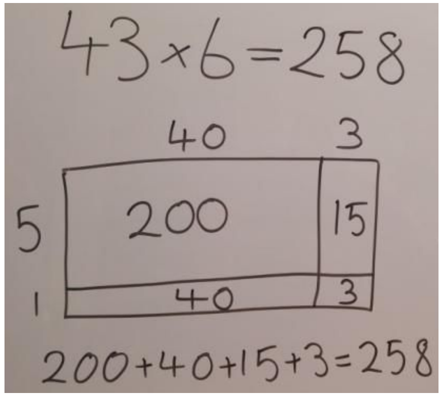
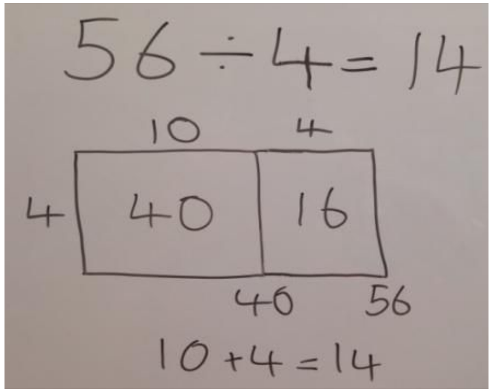
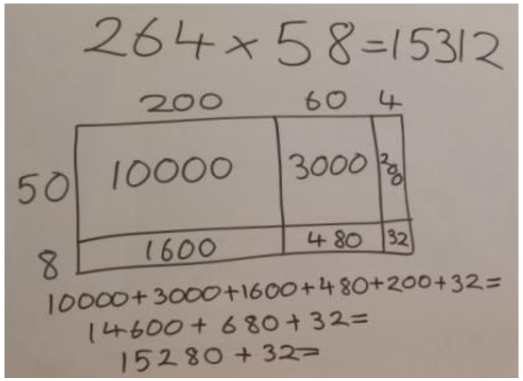
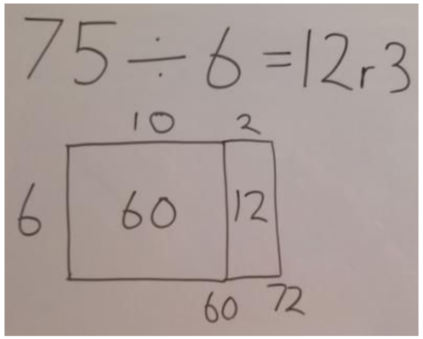
Stage	Skill, concept or approach	Examples
Development of early skills	Early multiplication and division will explore 'groups of' and 'sharing'. This may not even be maths based	Can you sort the cars into colours? If you and your friend have 4 sweets, how many sweets will you each have so that it is fair?
Number lines and repeated addition	Multiplication and division are effectively calculated on an empty number line by framing as repeated addition or subtraction.	 
Partitioning and grouping (alongside number lines)	Use partitioning and grouping to develop flexibility and increase efficiency of the approach	  

Arrays	Arrays are representations that can be counted to find products or created by the method of chunking to solve division questions.	 
Grid	Partitioning the numbers in a multiplication question creates a grid that can be solved to give the answer. For division, a grid is created using known facts until the first number is made and the answer presents itself. Using a grid where size is proportionate is best practice and estimation can support successful answers.	   

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="3">When choosing the methods for calculating with multiplication, it is important to consider how the method helps you to understand the distributivity and commutativity of multiplication and whether the chosen method aids the calculation itself. This is the reason for placing an emphasis on the methods shown in the main body of the calculations policy. When using the formal algorithms for multiplication, identifying the distributivity and commutativity is not always possible. For this reason, it is important that the children understand the place value of the digits, what they are calculating and why at each stage of the algorithm.</td></tr>
<tr><td>Short multiplication</td><td></td><td><u>Short Multiplication:</u> 342 x 7 =

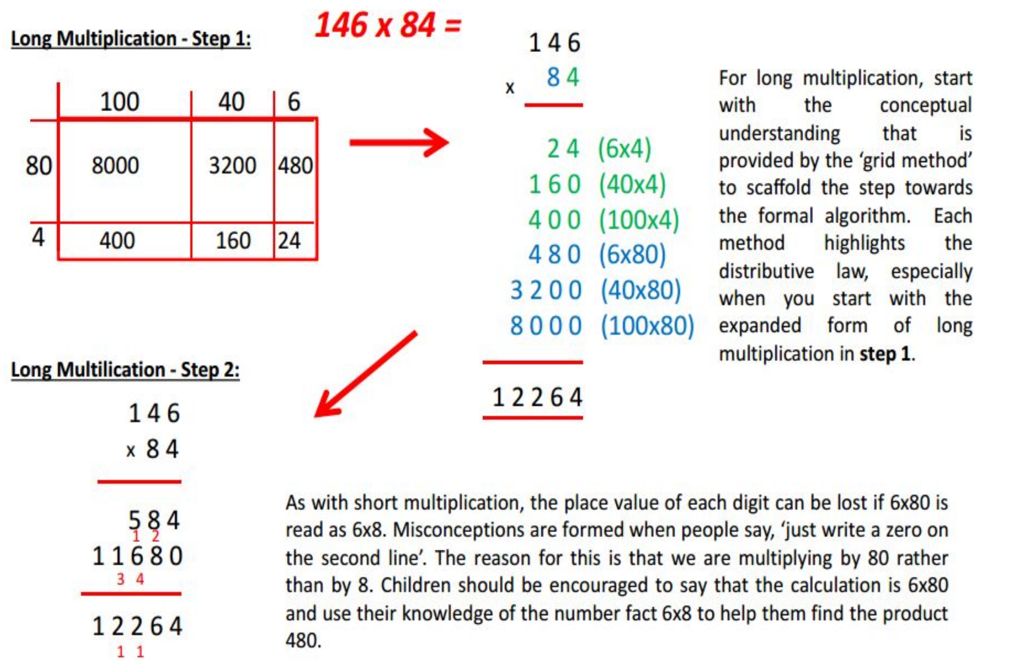
This method highlights the distributive law of multiplication as, like the grid method, you have to separate the component parts of the number to perform the calculation. The misconceptions can occur when 40x7 becomes 4x7. The place value of the digits needs to be preserved.

342
x 7
14 (2x7)
280 (40x7)
2100 (300x7)
2394

→

342
x 7
2394
2 1

As addition, short multiplication should start in an expanded form, before progressing to the compact version.</td></tr>
</table>

Long multiplication		

As with multiplication, when choosing calculation methods for division, there are certain considerations to take into account. Does the method show that division is not commutative? Does it highlight that you can swap the divisor and the quotient, but you cannot swap the divisor and the dividend? Does the method aid calculation? Using arrays and the number line do highlight these issues, however with the bus stop method, it can be less clear. Children should know how the bus stop methods for short and long division work, but are not expected to use them as their default method of calculation. To promote understanding of division and the calculation that is being performed, children should be encouraged to use the number line when dividing.

These methods should be learned so that children can answer questions in the Key Stage 2 assessments.

Bus stop division		**Short Division** 432 ÷ 5 $\begin{array}{r} 8\;\,6\;\text{r}2 \\ 5\overline{)4\;\,3\;{}^{3}2} \end{array}$ Answer = 86 r 2 or $86\frac{2}{5}$ or 86.4 432 ÷ 5 in isolation is a difficult calculation, mentally partitioning the number by place value in short division can help. You could calculate 400 ÷ 5 = 80, however this is unhelpful as recording the 80 would require writing the 8 in the tens column and ignoring the 0, which could be confusing. For this reason, calculate 43 ÷ 5, which gives 8, and then carry the 3 to make 32. This can then be divided by 5 to equal the 6 with 2 remainders. The remainder can be left as it is, or divided by 5 to give ⅖ or 0.4.